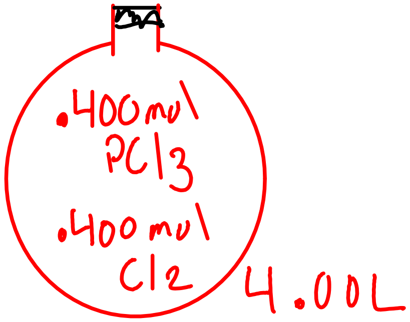
$$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g) \quad K_c = 49$$

If you add 0.400 moles of each reactant to a 4.00 L reaction vessel, what is the concentration of each species in the equilibrium mixture?



$$K_c = \frac{[PCl_5]}{[PCl_3][Cl_2]} = 49$$

We need to relate these concentrations to one another! Use a chart!

Initial conditions

Species	[Initial]	Δ	[Equilibrium]
PCl_5	0	$+x$	x
PCl_3	$\frac{0.400\ mol}{4.00\ L} = 0.100\ M$	$-x$	$0.100 - x$
Cl_2	$\frac{0.400\ mol}{4.00\ L} = 0.100\ M$	$-x$	$0.100 - x$

Let "x" equal the change in phosphorus pentachloride concentration

Plug back into the equilibrium expression...

$$\frac{(x)}{(0.100 - x)(0.100 - x)} = 49$$

Now we need to solve for "x"!

$$\frac{(x)}{(0.100-x)(0.100-x)} = 49$$

$(a-b)^2 = a^2 - 2ab + b^2$

$$\frac{x}{(0.0100 - 0.200x + x^2)} = 49$$

$x = 49(0.0100 - 0.200x + x^2)$

$x = 0.49 - 9.8x + 49x^2$

$0 = 0.49 - 10.8x + 49x^2$

$c = 0.49 \quad b = -10.8 \quad a = 49$

$$x = \frac{-(-10.8) \pm \sqrt{(-10.8)^2 - 4(49)(0.49)}}{2(49)} = \frac{10.8 \pm \sqrt{20.6}}{98}$$

$x = 0.157$ OR 0.0639

A quadratic equation has two mathematical solutions, but ...

The QUADRATIC EQUATION:

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Each quadratic has two solutions (see the +/- part of the equation), but only one of them will be the correct chemical solution.

Chemically, only one of these solutions is the correct one! How to choose!

Species	[Initial]	Δ	[Equilibrium]
PCl_5	0	+X	X
PCl_3	$\frac{0.400\ mol}{4.00\ L} = 0.100\ M$	−X	0.100 − X
Cl_2	$\frac{0.400\ mol}{4.00\ L} = 0.100\ M$	−X	0.100 − X

To figure out the correct solution, try plugging both possible values back into our chart...

$x = 0.157$ OR 0.0639

This solution would give negative concentrations for both phosphorus trichloride and chlorine. Negative concentrations are physically impossible, so we throw out this solution.

$[PCl_5] = x = 0.0639\ M\ PCl_5$

$[PCl_3] = 0.100 - x = 0.0361\ M\ PCl_3$

$[Cl_2] = 0.100 - x = 0.0361\ M\ Cl_2$

Since the equilibrium constant is fairly large (Kc = 49), we expect the equilibrium mix to have more products than reactants.

An 8.00 L reaction vessel at 3900C is charged with 0.850 mol of nitrogen and oxygen gases. Find the concentration of all species at equilibrium.

$$N_2(g) + O_2(g) \rightleftharpoons 2NO(g) \quad K_c = 0.0123$$

$$K_c = \frac{[NO]^2}{[N_2][O_2]} = 0.0123$$

We'll start by making a chart to relate these concentrations together.

Species	[Initial]	Δ	[Equilibrium]
N_2	$\frac{0.850\,mol}{8.00\,L} = 0.10625\,M$	$-x$	$0.10625 - x$
O_2	$\frac{0.850\,mol}{8.00\,L} = 0.10625\,M$	$-x$	$0.10625 - x$
NO	0	$+2x$	$2x$

Let "x" equal the change in nitrogen concentration

Plug back into the equilibrium expression ...

$$\frac{(2x)^2}{(0.10625 - x)(0.10625 - x)} = 0.0123$$

$$\frac{(2x)^2}{(0.10625-x)(0.10625-x)} = 0.0123$$

This is a second order equation. We can use the quadratic formula to solve it. But ...

$$\frac{(2x)^2}{(0.10625-x)^2} = 0.0123$$

Since the entire left hand side is a squared term, we can solve this one a little easier by taking the square root of both sides...

$$\sqrt{\frac{(2x)^2}{(0.10625-x)^2}} = \sqrt{0.0123}$$

Species	[Equilibrium]
N_2	$0.10625 - x$
O_2	$0.10625 - x$
NO	$2x$

$$\frac{2x}{0.10625-x} = 0.110905365$$

$$2x = 0.1109053651(0.10625-x)$$

$$2x = 0.011783695 - 0.1109053651x$$

$$2.1109053651x = 0.011783695$$

$$x = 0.00558$$

$[N_2] = 0.10625 - x = 0.101\ M\ N_2$

$[O_2] = 0.10625 - x = 0.101\ M\ N_2$

$[NO] = 2x = 0.0112\ M\ NO$