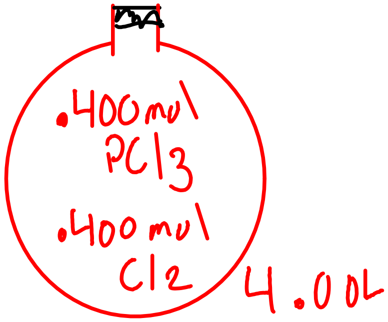
$$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g) \quad K_c = 49$$

If you add 0.400 moles of each reactant to a 4.00 L reaction vessel, what is the concentration of each species in the equilibrium mixture?



Initial conditions

$$K_c = \frac{[PCl_5]}{[PCl_3][Cl_2]} = 49$$

There are three concentration in the equilibrium expression which we need to solve. We'll have to relate them to each other somehow!

Species	[Initial]	Δ	[Equilibrium]
PCl_5	0	+X	0 + x = X
PCl_3	$\frac{0.400\ mol}{4.00\ L} = 0.100$	−X	0.100 − X
Cl_2	$\frac{0.400\ mol}{4.00\ L} = 0.100$	−X	0.100 − X

Let "x" equal the change in concentration of phosphorus pentachloride!

Substitute (Equilibrium) expressions into the Kc equation ...

$$\frac{(x)}{(0.100 - x)(0.100 - x)} = 49$$

If we can solve this equation for "x", we can find all the concentrations we want!

$$\frac{x}{(0.100-x)(0.100-x)} = 49$$

$$x = 49(0.100-x)^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$x = 49(0.01 - 0.200x + x^2)$$

$$x = 0.49 - 9.8x + 49x^2$$

$$0 = 0.49 - 10.8x + 49x^2$$

$c = 0.49$ $b = -10.8$ $a = 49$

$$x = \frac{10.8 \pm \sqrt{(-10.8)^2 - 4(49)(0.49)}}{2(49)} = \frac{10.8 \pm \sqrt{20.6}}{98}$$

$x = 0.157$ or $x = 0.0639$

One of these answers will be chemically impossible! Let's check!

The QUADRATIC EQUATION:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Each quadratic has two solutions (see the +/- part of the equation), but only one of them will be the correct chemical solution.

The x=0.157 solution causes the concentrations of both phosphorus trichloride and chlorine gas to go negative. This isn't possible (conservation of mass says we can't use more material than we have), so the correct x must be x=0.0639 ...

We can find the concentrations we're asked for by plugging x=0.0639 into our (Equilibrium) expressions we devised at the beginning --->

Species	[Equilibrium]
PCl_5	$0 + x = x$
PCl_3	$0.100 - x$
Cl_2	$0.100 - x$

$[PCl_5] = x = 0.064\ M\ PCl_5$

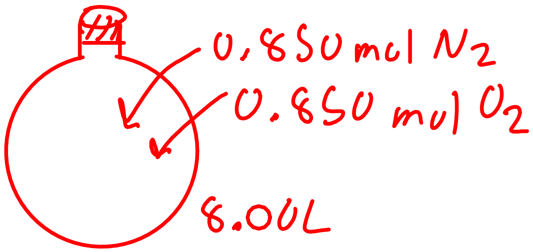
$[PCl_3] = 0.100 - x = 0.036\ M\ PCl_3$

$[Cl_2] = 0.100 - x = 0.036\ M\ Cl_2$

Final check: The value of Kc was 49 ... which means that we expect more products than reactants in our equilibrium mixture. That's consistent with the concentrations we calculated above, so our math is probably good!

114 An 8.00 L reaction vessel at 3900C is charged with 0.850 mol of nitrogen and oxygen gases. Find the concentration of all species at equilibrium.

$$N_2(g) + O_2(g) \rightleftharpoons 2NO(g);\ K_c = 0.0123$$



$$K_c = \frac{[NO]^2}{[N_2][O_2]} = 0.0123$$

As before. we need to reduce the number of variables..

Initial conditions...

Species	[Initial]	Δ	[Equilibrium]
N_2	$\frac{0.850\,mol}{8.00\,L} = 0.10625$	$-x$	$0.10625 - x$
O_2	$\frac{0.850\,mol}{8.00\,L} = 0.10625$	$-x$	$0.10625 - x$
NO	0	$+2x$	$2x$

Let "x" equal the loss in concentration of nitrogen gas...

$$\frac{(2x)^2}{(0.10625 - x)(0.10625 - x)} = 0.0123$$

We'll need to solve this equation for "x" to get our concentrations...

$$\frac{(2x)^2}{(0.10625-x)(0.10625-x)} = 0.0123$$

This is a quadratic! But we have other options to solve this equation...

$$\sqrt{\frac{(2x)^2}{(0.10625-x)^2}} = \sqrt{0.0123}$$

Since the entire left side is a squared term and the right is just a number, let's take the square root of each side!

Species	[Equilibrium]
N_2	$0.10625-x$
O_2	$0.10625-x$
NO	$2x$

$$\frac{2x}{0.10625-x} = 0.110905365I$$

$$2x = 0.110905365I(0.10625-x)$$

$$2x = 0.01178369S - 0.110905365Ix$$

$$2.110905365Ix = 0.01178369S$$

$$x = 0.0055822943$$

Now, plug into the (Equilibrium) expressions to find concentration...

$[N_2] = 0.10625 - x = 0.101\ M\ N_2$

$[O_2] = 0.10625 - x = 0.101\ M\ O_2$

$[NO] = 2x = 0.0112\ M\ NO$