

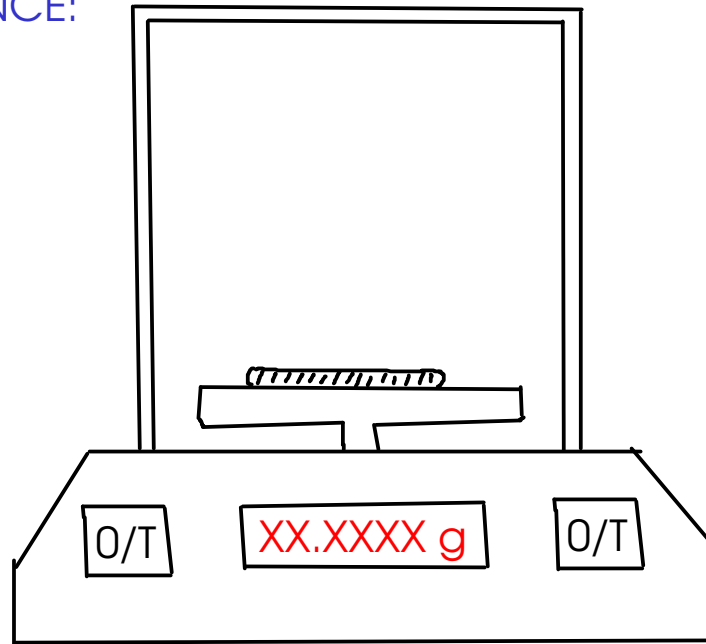
## More on precision

Every measurement contains some amount of ERROR, or some amount of deviation from the true value of what is being measured.

RANDOM ERROR is the variability in a measurement that cannot be traced back to a single cause. Random errors cause measurements to fluctuate around the true value, but can be averaged out given enough measurements.

When reporting measurements, we want to indicate how much random error we think is present. How?

We'll go to the lab and measure the mass of a metal ring using an ANALYTICAL BALANCE:



## Our classroom experiment: Results

| Mass of ring (g) |  |
|------------------|--|
| 42.1060          |  |
| 42.1057          |  |
| 42.1064          |  |
| 42.1060          |  |
| 42.1057          |  |
| 42.1059          |  |
| 42.1059          |  |
| 42.1061          |  |
| 42.1065          |  |
| 42.1062          |  |
| 42.1061          |  |

(11 measurements)

Overall average

$$\bar{x} = 42.1060454545 \text{ g (unrounded)}$$

$$= \underline{42.1060} \text{ g } (\pm 0.0001 \text{ g})$$

CERTAIN DIGITS: Appear in nearly all repeats of the measurement

UNCERTAIN DIGITS: Vary.. Variation caused by estimation or other sources of random error.

When reading measurements from a scale, record all CERTAIN digits (read directly from scale) and one UNCERTAIN (or estimated) digit.

When using a digital device, record all the displayed digits.

## Significant figures

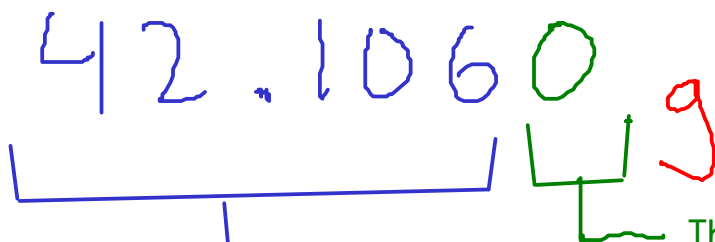
SIGNIFICANT FIGURES are a way to indicate the amount of uncertainty in a measurement.

The significant figures in a measurement are all of the CERTAIN DIGITS plus one and only one UNCERTAIN (or estimated) DIGIT

Example:

From our classroom experiment,

42.1060 g



This digit was estimated by the balance (it varied between measurements). This is an UNCERTAIN digit.

These digits were obtained in nearly all measurements of the ring. They are CERTAIN.

**THIS AVERAGE HAS SIX SIGNIFICANT FIGURES!**

## Determining significant figures

When you read a measurement that someone has written using the significant figures convention, you can tell how precisely that measurement was made.

$$1.47\text{(3)} \text{ g} \pm 0.001$$

This was measured to the nearest +/- 0.001 g  
The last digit is always UNCERTAIN (or estimated)

$$2\text{(1)} \text{ m} \pm 1$$

$$37.2\text{(6)} \text{ kg} \pm 0.01$$

Some other examples

$$3.207\text{(6)} \text{ g} \pm 0.0001\text{g}$$

$$27.3\text{(1)} \text{ m} \pm 0.1\text{m}$$

## A small problem

The number ZERO has several uses. It may be a measured number, but it may also be a mere "placeholder" that wasn't measured at all!

So how do we tell a measured zero from a placeholder? There are a few ways:

1: BEGINNING ZEROS: Beginning zeros are NEVER considered significant.

0.15 g

• 15 g  
↑

This zero merely indicates that there is a decimal point coming up!

0.015 m (1.5 cm)

These zeros are placeholders. They'll disappear if you change the UNITS of this number!

0,00063 mm

None of these zeros are considered significant

2: END ZEROS are sometimes considered significant. They are significant if

- there is a WRITTEN decimal point in the number
- there is another written indicator that the zero is significant. Usually this is a line drawn over or under the last zero that is significant!

$1.50 \text{ km} \pm 0.01 \text{ km}$

This zero IS considered significant. There's a written decimal.

$1500 \text{ m} \pm 100 \text{ m}$

These zeros ARE NOT considered significant (no written decimal, and no other indication that the zeros are significant)

$143\bar{0}00 \text{ g} \pm 100 \text{ g}$

These zeros are not significant.

This zero IS significant. It's marked.



## Calculations with measurements

When you calculate something using measured numbers, you should try to make sure the ANSWER reflects the quality of the data used to make the calculation.

An ANSWER is only as good as the POOREST measurement that went into finding that answer!

$$\begin{array}{r}
 14.206 \quad \pm 0.001 \\
 154.72 \quad \pm 0.01 \\
 1.6 \quad \pm 0.1 \\
 + 0.222 \quad \pm 0.001 \\
 \hline
 170.748
 \end{array}$$

How should we report this answer? How much uncertainty is in this answer?

$$170.7$$

- \* If you add an uncertain number to either a certain or an uncertain number, then the result is uncertain!
- \* If you add certain numbers together, the result is certain!



For addition and subtraction, round FINAL ANSWERS to the same number of decimal places as the measurement with the fewest decimal places. This will give an answer that indicates the proper amount of uncertainty.

For multiplication and division, round FINAL ANSWERS to the same number of SIGNIFICANT FIGURES as the measurement with the fewest SIGNIFICANT FIGURES!

$$\overset{4}{\underline{15.62}} \times \overset{3}{\underline{0.0667}} \times \overset{3}{\underline{35.0}} = 36.46489$$

How should we report this answer?

36.5

$$\overset{3}{\underline{25.4}} \times \overset{2}{\underline{0.00023}} \times \overset{5}{\underline{15.201}} = 0.088804242$$

How should we report this answer?

0.089

The first significant figure in this one is the first "8" (since lead zeros are NOT significant).

A few more math with significant figures examples:

$$\begin{array}{c} \underline{5} \\ 15047 \end{array} \times \begin{array}{c} \underline{2} \\ 11 \end{array} \times \begin{array}{c} \underline{4} \\ 0.9876 \end{array} = 163464.5892$$

~~16~~

~~160000.0000~~

160000

$1.6 \times 10^5$

Placeholder zeroes (or scientific notation) required here since we need to know where the decimal goes!

Addition:

$$\begin{array}{r} 147.3 \quad \pm 0.1 \\ 2432 \quad \pm 1 \\ 0.97 \quad \pm 0.01 \\ + 111.6 \quad \pm 0.1 \\ \hline 2691.87 \end{array}$$

2692

DENSITY  
CALCULATION

$$\begin{array}{r} \overbrace{14.7068}^6 \text{ g} \\ \hline \underbrace{2.7}_{-2} \text{ mL} \\ \hline = 5.446962963 \text{ g/mL} \end{array}$$

5.4 g/mL

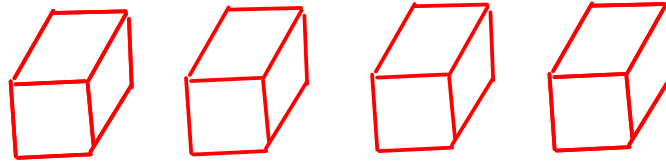
The only way to improve the precision of this density measurement is to improve the precision of the VOLUME measurement, since it limits the precision of the answer.

(We can actually use a LESS precise balance than the one we're currently using and still have the same quality density measurement!)

## Exact Numbers

- Some numbers do not have any uncertainty. In other words, they weren't measured!

1) Numbers that were determined by COUNTING!



How many blocks are to the left?  
exactly 4

2) Numbers that arise from DEFINITIONS, often involving relationships between units

$$12 \text{ in} = 1 \text{ ft}$$
$$\text{km} = 10^3 \text{ m}$$

\* All metric prefixes  
are exact!

- Treat exact numbers as if they have INFINITE significant figures or decimal places!

Example

You'll need to round the answer to the right number of significant figures!

Convert 4.45 m to in, assuming that  $2.54 \text{ cm} = 1 \text{ in}$

EXACT!

$$2.54 \text{ cm} = 1 \text{ in} \quad 1 \text{ cm} = 10^{-2} \text{ m}$$

$$4.45 \text{ m} \times \frac{1 \text{ cm}}{10^{-2} \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} = 175.1968504 \text{ in}$$

$$= \boxed{175 \text{ in}}$$

Significant figures analysis:  
 - 4.45 m: 3 significant figures (indicated by an upward arrow and the number 3)  
 -  $10^{-2} \text{ m}$ : infinite significant figures (indicated by an upward arrow and the symbol  $\infty$ )  
 - 2.54 cm: infinite significant figures (indicated by an upward arrow and the symbol  $\infty$ )

Usually, in unit conversions the answer will have the same number of significant figures as the original measurement did.

EXCEPTION: Temperature conversions, since these often involve ADDITION (different rule!)

A note on rounding: If possible, try to round only at the END of a multiple-step calculations. Avoid rounding intermediate numbers if possible, since extra rounding introduces ERROR into your calculations.