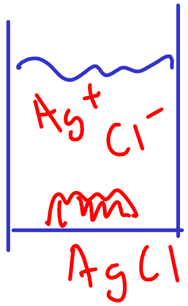
EXAMPLE: Calculate the grams per liter of silver(i) chloride (AgCl) in a solution that is at equilibrium with solid AgCl.

$$AgCl(s) \rightleftharpoons Ag^+(aq) + Cl^-(aq) \; ; \; K_c = 1.8 \times 10^{-10}$$



$$K_c = [Ag^+][Cl^-] = 1.8 \times 10^{-10}$$

Assign a variable, 'x', to equal the change in concentration of silver(I) ion...

Species	[Initial]	Δ	[Equilibrium]
Ag^+	0	+X	X
Cl^-	0	+X	X

Each time we make a silver(I) ion, we also make a chloride ion. (In the chemical equation, there's a 1:1 ratio of silver(I) to chloride)

$$[Ag^+][Cl^-] = 1.8 \times 10^{-10}$$

$$(x)(x) = 1.8 \times 10^{-10}$$

$$x = 1.34 \times 10^{-5} \; ; \; [Ag^+] = [Cl^-] = 1.34 \times 10^{-5} \, M$$

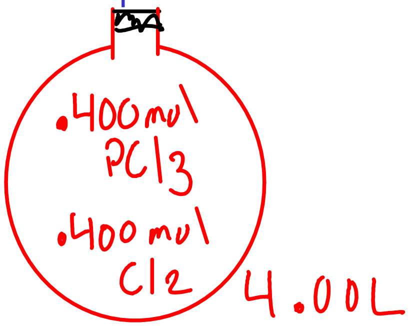
The concentration of dissolved AgCl EQUALS the concentration of either silver or chloride ion (which we know)...

$$[AgCl]_{dissolved} = 1.34 \times 10^{-5} \frac{mol}{L} \times \frac{143.35 \, g \, AgCl}{mol \, AgCl} = \boxed{0.0019 \, g/L}$$

Equivalent to a concentration of 1.9 ppm (parts per million) Equal to mg/L for dilute aqueous solutions!

$$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g) \quad K_c = 49$$

If you add 0.400 moles of each reactant to a 4.00 L reaction vessel, what is the composition of the equilibrium mixture?



$$K_c = \frac{[PCl_5]}{[PCl_3][Cl_2]} = 49$$

These concentrations are molar concentrations AT EQUILIBRIUM!

Initial conditions

Species	Initial	Δ	Equilibrium
PCl_3	$\frac{0.400\ mol}{4.00\ L} = 0.100\ M$	$-x$	$0.100 - x$
Cl_2	$\frac{0.400\ mol}{4.00\ L} = 0.100\ M$	$-x$	$0.100 - x$
PCl_5	0	$+x$	x

We've defined 'x' to be the change in concentration of phosphorus trichloride!

$$\frac{[PCl_5]}{[PCl_3][Cl_2]} = \frac{x}{(0.100 - x)(0.100 - x)} = 49$$

To solve the problem, we must solve this expression for 'x'

$$\frac{x}{(0.100-x)(0.100-x)} = 49$$

$$\frac{x}{(0.100-x)^2} = 49$$

$$x = 49(0.100-x)^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$x = 49(0.0100 - 0.200x + x^2)$$

$$x = 0.49 - 9.8x + 49x^2$$

$$0 = 0.49 - 10.8x + 49x^2$$ ← Quadratic form!

$a = 49 \quad b = -10.8 \quad c = 0.49$

$$x = \frac{+10.8 \pm \sqrt{(-10.8)^2 - 4(49)(0.49)}}{2(49)} = \frac{10.8 \pm \sqrt{20.6}}{98}$$

$x = \sout{0.157}$ or 0.0639

This value of 'x' is CHEMICALLY impossible, since it would cause the concentrations of chlorine and phosphorus trichloride to be negative!

This equation is second order in 'x'. ... or in other words, it;s a QUADRATIC EQUATION:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Each quadratic has two solutions (see the +/- part of the equation), but onlu one of them will be the correct chemical solution.

Species	Initial	Δ	Equilibrium
PCl_3	$\frac{0.400\ mol}{4.00\ L} = 0.100\ M$	$-x$	$0.100 - x$
Cl_2	$\frac{0.400\ mol}{4.00\ L} = 0.100\ M$	$-x$	$0.100 - x$
PCl_5	0	$+x$	x

$x = 0.0639$

MOLAR CONCENTRATIONS — MOLES AT EQUILIBRIUM

$[PCl_3] = 0.100 - x = 0.036\ M \quad \times 4.00\ L = 0.144\ mol\ PCl_3$

$[Cl_2] = 0.100 - x = 0.036\ M \quad \times 4.00\ L = 0.144\ mol\ Cl_2$

$[PCl_5] = x = 0.064\ M \quad \times 4.00\ L = 0.256\ mol\ PCl_5$

Comparison of initial state and equilibrium state:

Initial		Equilibrium
0.400 mol PCl_3	→	0.144 mol PCl_3
0.400 mol Cl_2		0.144 mol Cl_2
0 mol PCl_5		0.256 mol PCl_5

122 An 8.00 L reaction vessel at 3900C is charged with 0.850 mol of nitrogen and oxygen gases. Find the concentration of all species at equilibrium.

$$N_2(g) + O_2(g) \rightleftharpoons 2NO(g) \quad K_c = 0.0123$$

$$K_c = 0.0123 = \frac{[NO]^2}{[N_2][O_2]}$$

To solve this, we must express all three concentrations in terms of a single variable.

Species	[Initial]	Δ	[Equil]
N_2	$\frac{0.850\text{ mol}}{8.00\text{ L}} = 0.10625$	$-x$	$0.10625 - x$
O_2	$\frac{0.850\text{ mol}}{8.00\text{ L}} = 0.10625$	$-x$	$0.10625 - x$
NO	0	$+2x$	$2x$

We let 'x' equal the change in concentration of nitrogen gas.

Tip - try to pick 'x' to be a change in concenatration of a species with a coefficient of '1'...

$$\frac{[NO]^2}{[N_2][O_2]} = \frac{(2x)^2}{(0.10625 - x)(0.10625 - x)} = 0.0123$$

We need to solve this expression for 'x'

$$\frac{(2x)^2}{(0.10625-x)(0.10625-x)} = 0.0123$$

$$\sqrt{\frac{(2x)^2}{(0.10625-x)^2}} = \sqrt{0.0123}$$

You can solve this equation with the quadratic equation, or you can take the square root of both sides ...

$$\frac{2x}{0.10625-x} = 0.110905365$$

$$2x = (0.110905365)(0.10625-x)$$

$$18.0333926x = 0.10625-x$$

$$19.0339269x = 0.10625$$

$$x = 0.005582243$$

Now, use 'x' to solve for the concentrations!

Species	[Equil]
N_2	$0.10625-x$
O_2	$0.10625-x$
NO	$2x$

N_2: $0.10625-x$ = $0.101\ M\ N_2$

O_2: $0.10625-x$ = $0.101\ M\ O_2$

NO: $2x$ = $0.0112\ M\ NO$