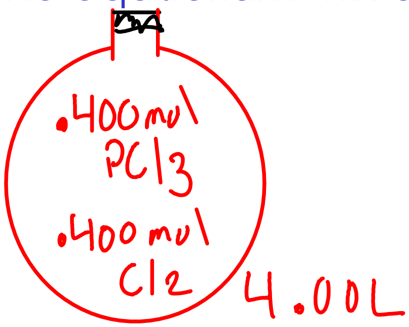
$$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g) \quad K_c = 49$$

If you add 0.400 moles of each reactant to a 4.00 L reaction vessel, what is the composition of the equilibrium mixture?




Initial conditions

Start with equilibrium expression:

$$K_c = \frac{[PCl_5]}{[PCl_3][Cl_2]} = 49$$

These concentrations are molar concentrations AT EQUILIBRIUM!

| Species | [Initial] | Δ | [Equilibrium] |
|---|---|---|---|
| $PCl_3$ | $\frac{0.400\text{ mol}}{4.00\text{ L}} = 0.100\text{ M}$ | $-x$ | $0.100 - x$ |
| $Cl_2$ | $\frac{0.400\text{ mol}}{4.00\text{ L}} = 0.100\text{ M}$ | $-x$ | $0.100 - x$ |
| $PCl_5$ | 0 M | $+x$ | $x$ |

We've defined 'x' as the concentration of phosphorus trichloride consumed.

$$\frac{[PCl_5]}{[PCl_3][Cl_2]} = \frac{(x)}{(0.100 - x)(0.100 - x)} = 49$$

To solve this problem, we need to solve this expression for 'x'.

Rearrange this expression to make it easier to solve. Isolate 'x' if possible.

$$\frac{(x)}{(0.100-x)(0.100-x)} = 49$$

$$\frac{x}{(0.100-x)^2} = 49$$

$$x = 49(0.100-x)^2$$

$(a-b)^2 = a^2 - 2ab + b^2$

$$x = 49(0.0100 - 0.200x + x^2)$$

$$x = 0.49 - 9.8x + 49x^2$$

$$0 = 49x^2 - 10.8x + 0.49$$

$a = 49 \quad b = -10.8 \quad c = 0.49$

$$x = \frac{10.8 \pm \sqrt{(-10.8)^2 - 4(49)(0.49)}}{2(49)} = \frac{10.8 \pm \sqrt{20.6}}{98}$$

$x = \sout{0.157}$ or 0.0639

This equation is a QUADRATIC EQUATION:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Each quadratic equation has TWO solutions. However, only ONE of the two solutions makes chemical sense!

This value for 'x' results in negative concentrations for both phosphorus trichloride and chlorine gas at equilibrium. Since this is impossible, we must discard this solution!

| Species | [Initial] | Δ | [Equilibrium] |
|---|---|---|---|
| $PCl_3$ | $\frac{0.400 \text{ mol}}{4.00 \text{ L}} = 0.100\ M$ | $-x$ | $0.100 - x$ |
| $Cl_2$ | $\frac{0.400 \text{ mol}}{4.00 \text{ L}} = 0.100\ M$ | $-x$ | $0.100 - x$ |
| $PCl_5$ | $0\ M$ | $+x$ | $x$ |

$x = 0.0639\ M$

Equilibrium concentrations

Number of moles of each substance

$[PCl_3] = 0.100 - 0.0639 = 0.0361\ M \times 4.00\ L = 0.144 \text{ mol } PCl_3$

$[Cl_2] = 0.100 - 0.0639 = 0.0361\ M \times 4.00\ L = 0.144 \text{ mol } Cl_2$

$[PCl_5] = 0.0639 = 0.0639\ M \times 4.00\ L = 0.256 \text{ mol } PCl_5$

Quick comparison of initial and final states

| INITIAL STATE | | EQUILIBRIUM STATE |
|---|---|---|
| 0.400 mol $PCl_3$<br>0.400 mol $Cl_2$<br>0 mol $PCl_5$ | $K_c = 49$ → | 0.144 mol $PCl_3$<br>0.144 mol $Cl_2$<br>0.256 mol $PCl_5$ |

An 8.00 L reaction vessel at 3900C is charged with 0.850 mol of nitrogen and oxygen gases. Find the concentration of all species at equilibrium.

$$N_2(g) + O_2(g) \rightleftharpoons 2NO(g) \quad K_c = 0.0123$$

$$K_c = 0.0123 = \frac{[NO]^2}{[N_2][O_2]}$$

We need to express all of these concentrations in terms of a single variable.

| Species | [Initial] | $\Delta$ | [Equilibrium] |
|---|---|---|---|
| $N_2$ | $\frac{0.850\,mol}{8.00\,L} = 0.10625\,M$ | $-x$ | $0.10625 - x$ |
| $O_2$ | $\frac{0.850\,mol}{8.00\,L} = 0.10625\,M$ | $-x$ | $0.10625 - x$ |
| $NO$ | $0$ | $+2x$ | $2x$ |

Let 'x' be the change in concentration of nitrogen gas.

$$\frac{[NO]^2}{[N_2][O_2]} = \frac{(2x)^2}{(0.10625-x)(0.10625-x)} = 0.0123$$

We need to solve the above expression for 'x' to continue.

$$\frac{(2x)^2}{(0.10625-x)(0.10625-x)} = 0.0123$$

$$\sqrt{\frac{(2x)^2}{(0.10625-x)^2}} = \sqrt{0.0123}$$

Solve with quadratic OR simolify by taking square root of both sides!

$$\frac{2x}{0.10625-x} = 0.110905365\text{l}$$

$$2x = 0.011783695 - 0.1109053651x$$

$$2.1109053651x = 0.011783695$$

$$x = 0.0055822943$$

Now, use 'x' to calculate equilibrium concentrations:

$[N_2] = 0.10625 - x = 0.101\ \underline{M}$

$[O_2] = 0.10625 - x = 0.101\ \underline{M}$

$[NO] = 2x = 0.0112\ \underline{M}$

We know Kc = 0.0123 (small), so we expect reactants to dominate at equilibrium. (They do!)

| Species | [Equilibrium] |
|---|---|
| $N_2$ | 0.10625 − x |
| $O_2$ | 0.10625 − x |
| NO | 2x |