

## More on precision

Every measurement contains some amount of ERROR, or some amount of deviation from the true value of what is being measured.

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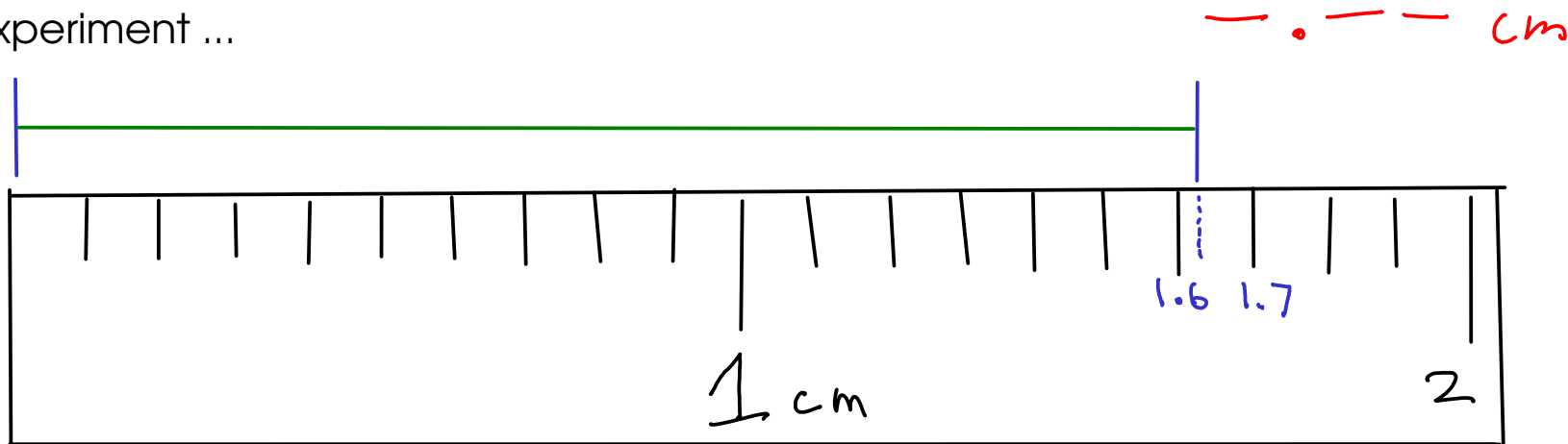
RANDOM ERROR is the variability in a measurement that cannot be traced back to a single cause. Random errors cause measurements to fluctuate around the true value, but can be averaged out given enough measurements.

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When reporting measurements, we want to indicate how much random error we think is present. How?

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An experiment ...



How long is the green line?

For this experiment, measure the line and record your answer in the form: X.XX cm  
(In other words, measure to the nearest 0.01 cm)

Write your answer on the card, then pass the card up to the front!

## Our classroom experiment: Results

After throwing away obvious mistakes in reading the scale, we had:

Value	# students
1.61	2
1.62	10
1.63	21
1.64	1

34 measurements

1.626176471 cm : unrounded average

Overall average

$$1.63 \pm 0.01 \text{ cm}$$

Certain.  
Little to no  
variation  
expected.  
Same almost  
every time

Uncertain.  
Expected to  
vary by about  
+/- 1

When reading measurements from a scale, record all CERTAIN digits and one UNCERTAIN (or estimated) digit.

## Significant figures

SIGNIFICANT FIGURES are a way to indicate the amount of uncertainty in a measurement.

The significant figures in a measurement are all of the CERTAIN DIGITS plus one and only one UNCERTAIN (or estimated) DIGIT

Example:

From our classroom experiment,

1.63 cm

We estimated the last digit, It's uncertain

These digits were obtained in all measurements. They are certain

THIS MEASUREMENT HAS "THREE SIGNIFICANT FIGURES"!

## Determining significant figures

When you read a measurement that someone has written using the significant figures convention, you can tell how precisely that measurement was made.

$$1.47\text{(3)} \text{ g} \pm 0.001\text{g}$$

← approximate uncertainty

This was measured to the nearest +/- 0.001 g  
The last digit is always UNCERTAIN (or estimated)

$$2\text{(1)} \text{ m} \pm 1\text{m}$$

$$37.2\text{(6)} \text{ kg} \pm 0.01\text{kg}$$

Some other examples

$$3.2076 \text{ g} \pm 0.0001\text{g}$$

$$27.3 \text{ m} \pm 0.1\text{m}$$

└─┬─┘  
uncertain digits

## A small problem

The number ZERO has several uses. It may be a measured number, but it may also be a mere "placeholder" that wasn't measured at all!

So how do we tell a measured zero from a placeholder? There are a few ways:

1: BEGINNING ZEROS: Beginning zeros are NEVER considered significant.

$0.15 \text{ g}$ 
(  $.15 \text{ g}$  ... another way of writing )  
 $0.15 \text{ g}$

This zero merely indicates that there is a decimal point coming up!

$0.015 \text{ m}$  ( $1.5 \text{ cm}$ )

These zeros are placeholders. They'll disappear if you change the UNITS of this number!

$0,00063 \text{ mm}$

None of these zeros are considered significant

2: END ZEROS are sometimes considered significant. They are significant if

- there is a WRITTEN decimal point in the number
- there is another written indicator that the zero is significant. Usually this is a line drawn over or under the last zero that is significant!

$1.50 \text{ km} \pm \underline{\underline{0.01 \text{ km}}}$ 
  
 (A green box highlights "1.50" with a red bracket under the "0". A blue arrow points from the underlined "0.01 km" to the text "approximate uncertainty".)

This zero IS considered significant. There's a written decimal.

$1500 \text{ m} \pm 100 \text{ m}$ 
  
 (A green box highlights "1500" with a red bracket under the two trailing zeros.)

These zeros ARE NOT considered significant (no written decimal, and no other indication that the zeros are significant)

$143\overline{0}00 \text{ g} \pm 100 \text{ g}$ 
  
 (A green box highlights "143000" with a red bracket under the last three zeros. A blue arrow points from the overlined "0" to the text "This zero IS significant. It's marked.")

These zeros are not significant.

This zero IS significant. It's marked.

How many significant figures are there in each of these measurements?

$$\frac{76.070 \text{ g}}{5} \pm 0.001 \text{ g}$$

$$\frac{85000. \text{ mm}}{3} \pm 1 \text{ mm}$$

↑  
decimal point

$$\frac{0.001030 \text{ kg}}{4} \pm 0.000001 \text{ kg}$$

$$\frac{156.0002 \text{ g}}{7} \pm 0.0001 \text{ g}$$

$$\frac{0.10 \text{ s}}{2} \pm 0.01 \text{ s}$$

$$\frac{17000000 \text{ mg}}{2} \pm 1,000,000 \text{ mg}$$

$$\frac{120\bar{0}00 \text{ km}}{4} \pm 100 \text{ km}$$

$$\frac{1350 \text{ ms}}{3} \pm 10 \text{ ms}$$

(Number of significant figures is indicated in RED below each measurement. Significant digits are UNDERLINED.)

(Approximate uncertainty in each of these measurements is indicated in GREEN after each one.)

## Calculations with measurements

When you calculate something using measured numbers., you should try to make sure the ANSWER reflects the quality of the data used to make the calculation.

An ANSWER is only as good as the POOREST measurement that went into finding that answer!

$$\begin{array}{r}
 14.206 \\
 154.72 \\
 1.6 \\
 + 0.222 \\
 \hline
 170.748
 \end{array}$$

Round so that there's only one uncertain digit in the answer!

How should we report this answer? How much uncertainty is in this answer?

$$170.7$$

- \* If you add an uncertain number to either a certain or an uncertain number, then the result is uncertain!
- \* If you add certain numbers together, the result is certain!



For addition and subtraction, round FINAL ANSWERS to the same number of decimal places as the measurement with the fewest decimal places. This will give an answer that indicates the proper amount of uncertainty.

For multiplication and division, round FINAL ANSWERS to the same number of SIGNIFICANT FIGURES as the measurement with the fewest SIGNIFICANT FIGURES!

$$\overset{4}{\underline{15.62}} \times \overset{3}{\underline{0.0667}} \times \overset{3}{\underline{35.0}} = 36.46489$$

How should we report this answer?

36.5

$$\overset{3}{\underline{25.4}} \times \overset{2}{\underline{0.00023}} \times \overset{5}{\underline{15.201}} = 0.088804242$$

How should we report this answer?

0.089

A few more math with significant figures examples:

$$\overset{5}{15047} \times \overset{2}{11} \times \overset{4}{0.9876} = 163464.5892 \quad \boxed{160000}$$

~~16~~

Placeholder zeros, even though they aren't SIGNIFICANT, still need to be included, so we know how big the number is!

$$\begin{array}{r} 147.3 \\ 2432 \\ 0.97 \\ + 111.6 \\ \hline 2691.87 \end{array}$$

$\boxed{2692}$

DENSITY  
CALCULATION

$$\begin{array}{r} \overset{6}{14.7068 \text{ g}} \\ \hline \underset{2}{2.7 \text{ mL}} \\ \hline = 5.446962963 \text{ g/mL} \end{array}$$

$\boxed{5.4 \text{ g/mL}}$

To improve (make more precise) this calculated density, we must improve the poorest measurement. We must use a more precise device to measure the VOLUME (which only has two significant figures in this example)!