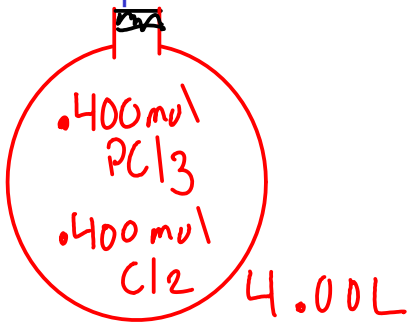
$$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g) \quad K_c = 49$$

If you add 0.400 moles of each reactant to a 4.00 L reaction vessel, what is the composition of the equilibrium mixture?



Initial conditions

Start with the equilibrium expression:

$$K_c = 49 = \frac{[PCl_5]}{[PCl_3][Cl_2]}$$

These concentrations are all EQUILIBRIUM concentrations!

SPECIES	INITIAL CONCENTRATION	Δ	EQUILIBRIUM CONCENTRATION
PCl_3	$\frac{0.400\,mol}{4.00\,L} = 0.100\,M$	$-x$	$0.100 - x$
Cl_2	$\frac{0.400\,mol}{4.00\,L} = 0.100\,M$	$-x$	$0.100 - x$
PCl_5	$0\,M$	$+x$	x

$$\frac{[PCl_5]}{[PCl_3][Cl_2]} = \frac{(x)}{(0.100-x)(0.100-x)} = 49$$

To solve the problem, we need to solve this expression for 'x'!

Rearrange the equation to make it easier to solve. In other words, try to isolate 'x' if possible.

$$\frac{(x)}{(0.100-x)(0.100-x)} = 49$$

$$\frac{x}{(0.100-x)^2} = 49$$

$$x = 49(0.100-x)^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$x = 49(0.0100 - 0.200x + x^2)$$

$$x = 0.49 - 9.8x + 49x^2$$

$$0 = 49x^2 - 10.8x + 0.49$$

$$a = 49 \quad b = -10.8 \quad c = 0.49$$

This equation is a QUADRATIC EQUATION:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Each quadratic equation has TWO solutions. However, only ONE of the two solutions makes chemical sense!

$$x = \frac{10.8 \pm \sqrt{(-10.8)^2 - 4(49)(0.49)}}{2(49)} = \frac{10.8 \pm \sqrt{20.6}}{98}$$

$x = 0.157$ OR 0.0639

This value for 'x' results in NEGATIVE concengtrations for both phosphorus trichloride and chlorine gas at equilibrium. This is not physically possible, so we throw out this solution.

	Initial	Δ	Equilibrium
$[PCl_3]$	$\frac{.400\ mol}{4.00\ L} = .100$	$-X$	$0.100 - X$
$[Cl_2]$	$\frac{.400\ mol}{4.00\ L} = .100$	$-X$	$0.100 - X$
$[PCl_5]$	0	$+X$	X

$X = 0.0639\ M$

$[PCl_3] = 0.100 - X = 0.0361\ M \times 4.00\ L = .144\ mol\ PCl_3$

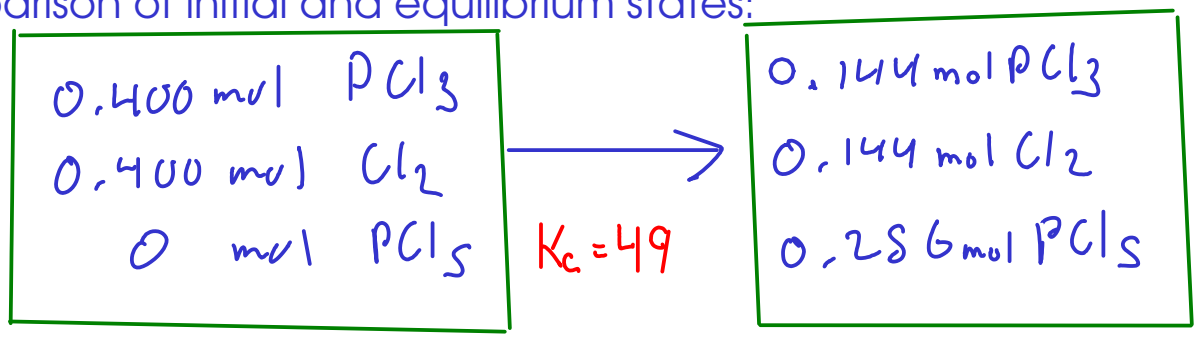
$[Cl_2] = 0.100 - X = 0.0361\ M \times 4.00\ L = .144\ mol\ Cl_2$

$[PCl_5] = X = 0.0639\ M \times 4.00\ L = .256\ mol\ PCl_5$

Concentrations at equilibrium

Number of moles in reaction vessel at equilibrium

Quick comparison of initial and equilibrium states:



INITIAL STATE

EQUILIBRIUM STATE