

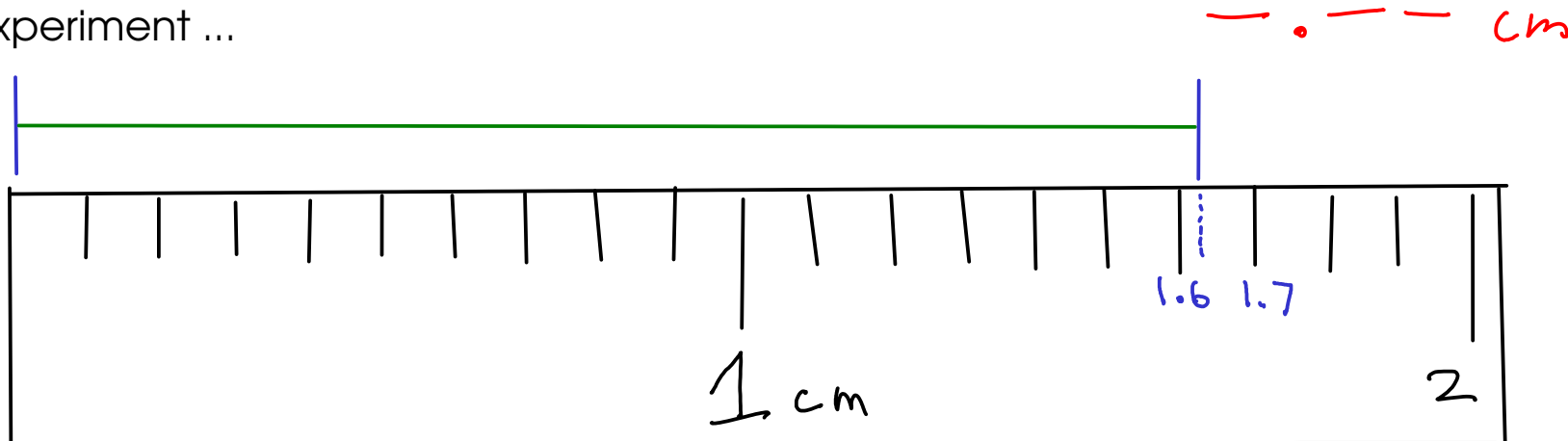
More on precision

Every measurement contains some amount of ERROR, or some amount of deviation from the true value of what is being measured.

RANDOM ERROR is the variability in a measurement that cannot be traced back to a single cause. Random errors cause measurements to fluctuate around the true value, but can be averaged out given enough measurements.

When reporting measurements, we want to indicate how much random error we think is present. How?

An experiment ...



How long is the green line?

For this experiment, measure the line and record your answer in the form: X.XX cm
(In other words, measure to the nearest 0.01 cm)

Write your answer on the card, then pass the card up to the front!

Our classroom experiment: Results

After throwing away obvious mistakes in reading the scale, we had:

Value	# students
1.62	4
1.63	3
1.64	1

8 total measurements

1.62625 cm : unrounded average

Overall average

$$1.63 \pm 0.01 \text{ cm}$$

Certain.

Little to no
variation
expected.

Same almost
every time

Uncertain.

Expected to
vary by about
+/- 1

When reading measurements from a scale, record all CERTAIN digits and one UNCERTAIN (or estimated) digit.

Significant figures

SIGNIFICANT FIGURES are a way to indicate the amount of uncertainty in a measurement.

The significant figures in a measurement are all of the CERTAIN DIGITS plus one and only one UNCERTAIN (or estimated) DIGIT

Example:

From our classroom experiment,

1.63 cm

1.6

3

cm

We estimated the last digit, It's uncertain

These digits were obtained in all measurements. They are certain

THIS MEASUREMENT HAS "THREE SIGNIFICANT FIGURES"!

Determining significant figures

When you read a measurement that someone has written using the significant figures convention, you can tell how precisely that measurement was made.

$$1.47\text{(3)} \text{ g} \pm 0.001 \text{ g}$$

← approximate uncertainty

This was measured to the nearest +/- 0.001 g
The last digit is always UNCERTAIN (or estimated)

$$2\text{(1)} \text{ m} \pm 1 \text{ m}$$

$$37.2\text{(6)} \text{ kg} \pm 0.01 \text{ kg}$$

Some other examples

$$3.207\text{(6)} \text{ g} \pm 0.0001 \text{ g}$$

$$27.\text{(3)} \text{ m} \pm 0.1 \text{ m}$$

uncertain digit

A small problem

The number ZERO has several uses. It may be a measured number, but it may also be a mere "placeholder" that wasn't measured at all!

So how do we tell a measured zero from a placeholder? There are a few ways:

1: BEGINNING ZEROS: Beginning zeros are NEVER considered significant.

0.15 g
(0.15 g ... another way of writing)
 0.15 g

This zero merely indicates that there is a decimal point coming up!

0.015 m (1.5 cm)

These zeros are placeholders. They'll disappear if you change the UNITS of this number!

$0,00063 \text{ mm}$

None of these zeros are considered significant

2: END ZEROS are sometimes considered significant. They are significant if

- there is a WRITTEN decimal point in the number
- there is another written indicator that the zero is significant. Usually this is a line drawn over or under the last zero that is significant!

$1.50 \text{ km} \pm 0.01 \text{ km}$

approximate uncertainty

This zero IS considered significant. There's a written decimal.

$1500 \text{ m} \pm 100 \text{ m}$

These zeros ARE NOT considered significant (no written decimal, and no other indication that the zeros are significant)

$143\overline{0}00 \text{ g} \pm 100 \text{ g}$

These zeros are not significant.

This zero IS significant. It's marked.

How many significant figures are there in each of these measurements?

$$\frac{76.070}{5} \text{ g} \pm 0.001 \text{ g}$$

$$\frac{85000.}{5} \text{ mm} \pm 1 \text{ mm}$$

↑
decimal point

$$0.001030 \text{ kg} \pm 0.000001 \text{ kg}$$

$$\frac{156.0002}{7} \text{ g} \pm 0.0001 \text{ g}$$

$$0.10 \text{ s} \pm 0.01 \text{ s}$$

$$\frac{17000000}{2} \text{ mg} \pm 1,000,000 \text{ mg}$$

$$\frac{120\bar{0}00}{4} \text{ km} \pm 100 \text{ km}$$

$$\frac{1350}{3} \text{ ms} \pm 10 \text{ ms}$$

(Number of significant figures is indicated in RED below each measurement. Significant digits are UNDERLINED.)

(Approximate uncertainty in each of these measurements is indicated in GREEN after each one.)

Calculations with measurements

When you calculate something using measured numbers, you should try to make sure the ANSWER reflects the quality of the data used to make the calculation.

An ANSWER is only as good as the POOREST measurement that went into finding that answer!

$$\begin{array}{r}
 14.206 \text{ Kg} \\
 154.72 \text{ Kg} \\
 + 1.6 \text{ Kg} \\
 + 0.222 \text{ Kg} \\
 \hline
 170.748 \text{ Kg}
 \end{array}$$

Round so that there's only one uncertain digit in the answer!

How should we report this answer? How much uncertainty is in this answer?

$$170.7 \text{ Kg}$$

- * If you add an uncertain number to either a certain or an uncertain number, then the result is uncertain!
- * If you add certain numbers together, the result is certain!

For addition and subtraction, round FINAL ANSWERS to the same number of decimal places as the measurement with the fewest decimal places. This will give an answer that indicates the proper amount of uncertainty.

For multiplication and division, round FINAL ANSWERS to the same number of SIGNIFICANT FIGURES as the measurement with the fewest SIGNIFICANT FIGURES!

$$\overset{4}{\underline{15.62}} \times \overset{3}{\underline{0.0667}} \times \overset{3}{\underline{35.0}} = 36.46489$$

How should we report this answer?

36.5

$$\overset{3}{\underline{25.4}} \times \overset{2}{\underline{0.00023}} \times \overset{5}{\underline{15.201}} = 0.088804242$$

How should we report this answer?

0.089

A few more math with significant figures examples:

$$\overset{5}{15047} \times \overset{2}{11} \times \overset{4}{0.9876} = 163464.5892 \quad \boxed{160000} \quad (1.6 \times 10^5)$$

~~16~~

Placeholder zeros, even though they aren't SIGNIFICANT, still need to be included, so we know how big the number is!

$$\begin{array}{r} 147.3 \\ 2432 \\ 0.97 \\ + 111.6 \\ \hline 2691.87 \end{array}$$

2692

DENSITY
CALCULATION

$$\begin{array}{r} \overset{6}{14.7068 \text{ g}} \\ \hline \underset{2}{2.7 \text{ mL}} \end{array}$$

$$= 5.446962963 \text{ g/mL}$$

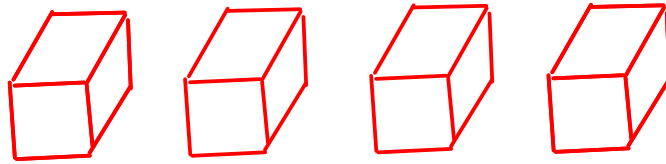
5.4 g/mL

To improve (make more precise) this calculated density, we must improve the poorest measurement. We must use a more precise device to measure the VOLUME (which only has two significant figures in this example)!

Exact Numbers

- Some numbers do not have any uncertainty. In other words, they weren't measured!

1) Numbers that were determined by COUNTING!



How many blocks are to the left?

exactly 4!

2) Numbers that arise from DEFINITIONS, often involving relationships between units

$$12 \text{ in} = 1 \text{ ft}$$

$$\text{km} = 10^3 \text{ m}$$

* All metric prefixes are exact!

- Treat exact numbers as if they have INFINITE significant figures!

35 Example

You'll need to round the answer to the right number of significant figures!

Convert 4.45 m to in, assuming that 2.54 cm = 1 in*

$$1 \text{ cm} = 10^{-2} \text{ m} \quad 2.54 \text{ cm} = 1 \text{ in}$$

$$\frac{4.45 \text{ m}}{10^{-2} \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} = 175.1968504 \text{ in}$$

↑ 3 ↑ ∞ ↑ ∞

Round the answer so that it has three SIGNIFICANT FIGURES, just like you do with any other multiplication/division using measurements!

$$= 175 \text{ in}$$

*An inch is defined as EXACTLY 2.54 cm !

When merely converting the units of a measurement, you almost always have the same number of significant figures in the answer as you did in the original measurement. (EXCEPTION: Temperature conversions, since they involve addition and subtraction)

Scientific Notation

- a way to represent large and small numbers
- a way to indicate significant figures

Form:

$$a.aad\dots \times 10^a$$

(always ONE nonzero digit before the decimal)

$$3.6 \times 10^4$$

means

$$3.6 \times 10 \times 10 \times 10 \times 10$$

OR

$$\underline{36000}$$

$$6.21 \times 10^{-3}$$

means

$$6.21 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$$

OR

$$\underline{0.00621}$$

Scientific notation removes the need for placeholder zeros, and that's good when you're dealing with very large and very small numbers!

$$4.70 \times 10^{-6} = 0.00000470$$

Scientific notation indicates significant figures without extra decimal points or lines. All numbers in front of the power of ten are significant!

$$3700 = 3.70 \times 10^3$$

To write a number in scientific notation, move the decimal point so that it is behind the first nonzero number. The power of ten will be the number of places you moved the decimal. If the number is less than 1, the power of ten is negative. If it's greater than one, the power of ten is positive.

$$0.00765$$

$$7.65 \times 10^{-3}$$

$$14000$$

$$1.400 \times 10^4$$

$$6.38 \times 10^5$$

Big number!

6.380000000000

$$638000$$

$$4.20 \times 10^{-6}$$

Small number!

000000000004.20

$$0.00000420$$

Using scientific notation on a calculator:

$$6.38 \times 10^5$$

on a TI-8x:

enter

$$6.38 \boxed{EE} 5$$

calculator displays:

$$6.38 E 5$$

this E means "x10 raised to"

$$4.20 \times 10^{-6}$$

enter:

$$4.20 \boxed{EE} \boxed{(-)} 6$$

calculator displays:

$$4.2 E^{-6}$$

means "x10⁻⁶"