## Introduction

Observations are vitally important to all of science. Some observations are qualitative in nature such as a change in color of a material, the formation of a gas, or the formation of solid in a liquid mixture. Many observations, though, are quantitative. You may measure how much something weighs, how much product is produced in a chemical reaction, the distance between two towns, etc. These quantitative observations - measurements - are done by comparing the thing being measured to some standard of measurement. This comparison introduces error, and it is for this reason that we must be careful when dealing with measurements.

## Measurements and units

All measurements are reported with units, which are fixed standards of measurement. Any quantitative measurement should be given with its associated unit. It's one thing to say Columbia, SC, is 82 miles from Florence, SC, but it's quite another to say that Florence is 82 inches away from Columbia. If I tell you Columbia is "82" away from here, you don't necessarily know what I mean. You'd probably guess miles - unless you were European and assumed I meant kilometers.

## Accuracy and precision

There are two things to worry about when making a measurement: the accuracy of the measurement and the precision of the measurement.

The accuracy of a measurement is the closeness of the measurement to the true value of the thing being measured. In other words - is the measurement right? You can check accuracy be measuring a standard - an object where the measured property has a known value. To check the accuracy for a scale, for example, you would measure a standard mass on the scale. If the scale measures a known mass correctly, it probably measures an unknown mass correctly.

The precision of a measurement is the reproducibility of the measurement. In other words, can you measure the same thing multiple times and get the same result? To check the precision of a measurement, we make multiple measurements.

How do real-world measurements rate when it comes to accuracy and precision? There are three common cases, which we will illustrate using a dart board.


In the case in Illustration 1, darts are scattered all over the dart board and nowhere near the bullseye. Obviously, the person throwing the darts is either a very poor shot or very drunk! In a lab, this would be a poor set of measurements done by an inexperienced or lazy experimenter. These measurements are neither accurate nor precise, and these measurements are worthless for providing evidence for a hypothesis.

| In this case, our measurements are |
| :--- |
| reproducible - they all have |
| approximately the same value. |
| The measurements, though, are not near |
| the true value. |

Illustration 2: Precise but not accurate.

In the case shown in Illustration 2, the measurements have approximately the same value, but are substantially "off" from the true value. These measurements are precise but not accurate. These measurements are not good evidence for a hypothesis. This case is most often caused by faulty or uncalibrated measuring devices. It is usually corrected by checking your measuring devices every once in a while using a known standard and calibrating when necessary.

| In this case, our measurements are |
| :--- |
| reproducible - they all have |
| approximately the same value. |
| The measurements are all near the true |
| value. |

Illustration 3: Accurate and precise

The case shown in illustration 3 is the best case - the measurements are all nearly the same and they are all approximately equal to the true value. These measurements are accurate and precise and are the kind of good data you need to back up a hypothesis.

## Precision and the concept of significant figures

Since reproducibility (precision) is so important when taking scientific measurements, we write our measurements in a way that indicates their precision - using a convention known as significant figures. To explain what significant figures are, we'll look at an example.


Graduated cylinders are common pieces of laboratory glassware. You can use them to measure out volumes of a liquid. You read a graduated cylinder by looking for the meniscus (the bottom of the curved surface of water) and comparing it to the scale on the cylinder.

Let's say that you were trying to read a graduated cylinder to the best of your ability. Try to read the cylinder drawn in illustration 5:

|  | - You may assume this cylinder is scaled such that each line represents one milliliter. <br> - Read from the meniscus. <br> - Read to one decimal place better than the scale - make an estimate of where the bottom surface of the water is located. |
| :---: | :---: |
|  |  |
| Illustration 5-A graduated cylinder containing liquid |  |

When reading this cylinder, you might come up with one of several answers: $15.3 \mathrm{~mL}, 15.5 \mathrm{~mL}$, $15.6 \mathrm{~mL}, 15.4 \mathrm{~mL}$, or 15.7 mL . It depends on how you estimate the bottom of the liquid. Some features all these numbers have in common is that you're certain that the cylinder contains 15 point something milliliters of liquid. There is some uncertainty in the last digit, since we estimated it.

Let's say that you read the cylinder as 15.4 mL . You would report the measurement as 15.4 mL . This number actually tells someone else how precisely you measured the volume. You know the first two digits in the number with certainty, while the last digit is your estimate (or "best guess") of how many tenths of a milliliter are in the cylinder. Significant figures are the certain digits in a measurement (the " 15 " in our example) plus one uncertain digit (the " 4 "). We say that a measurement of 15.4 mL contains "three significant figures".

It's easy to get the correct number of significant figures for a measurement:

1. For measurements where you read the markings on a scale, the markings indicate your certain digits, while your uncertain digit is one decimal place better than your markings.
2. For most measurements using digital readouts, take all the digits the measuring device
gives. This is less true for non-scientific equipment like cheap digital thermometers available in department stores.
3. For all measurements, you may make several measurements and find the standard deviation of the measurements. The standard deviation represents the uncertainty in a set of measurements. You will do this in one of your first laboratory experiments.

We have highly precise analytical balances in our lab to measure mass. If the balance reads 0.2442 g , that's how you would record the measurement. You might notice that the last digit on these balances may fluctuate slightly. The balance continually re-estimates the mass of an object on the balance, so you will sometimes see fluctuations.


## Math with significant figures

So how many significant figures do you report if you take multiple measurements (each with their own number of significant figures) and add / subtract / multiply / divide them? How can you tell how many significant figures are in a measurement if you didn't do it yourself?

Here are some rules of thumb for telling what digits are significant in the first place:

1. Any nonzero digit is significant.
2. Zeros at the end of a number are not considered significant unless there is a decimal
point in the number. The zero in " 5.0 cm " is significant - it means that the length was measured to a tench of a centimeter. The zeros is " 500 miles" aren't significant unless we're told otherwise. Did the experimenter measure to the nearest hundred miles, ten miles, or single mile?
3. Zeros at the beginning of a number (before any nonzero digit) are not significant. The zero in " 0.42 grams" is in front of the decimal and is put there simply to make the decimal easier to see. It is not significant. The three zeros in 0.0038 km are not significant, either.
4. If a measurement is written in scientific notation, all the digits in the mantissa are significant. For example, there are three significant digits in the measurement " $5.00 \times 10^{2}$ " miles - the one and the two zeros after the decimal point..

Non-significant zeros in a number are usually called placeholders - they exist to either show how big or small a number is, or to make the number easier to read. They are not part of the actual measurement.

Let's look at a few examples Significant digits will be underlined: $\backslash$

| Measurement | Significant figures | Explanation |
| :---: | :---: | :--- |
| $0.00 \underline{420} \mathrm{~cm}$ | 3 | The three beginning zeros aren't significant - they're <br> placeholders. The end zero is not a placeholde. |
| $\underline{123.456}$ miles | 6 | All nonzero digits are significant. |
| $\underline{102.0}$ seoonde | 4 | The first zero is significant - it was measured. The end <br> zero is after a decimal - it is not a placeholder. |
| $\underline{500}$ miles | 1 | Unless we're told otherwise, 500 miles must have been <br> measured to the nearest hundred miles. The zeros are <br> considered placeholders. |
| $0 . \underline{1200} \mathrm{~g}$ | 4 | The zero in front of the decimal is just a placeholder - it <br> makes the number easier to read by letting you know a <br> decimal point is coming up. |
| $\underline{1.230} \mathrm{x} 10^{-6} \mathrm{~kg}$ | 4 | The number is in scientific notation. |
| $\underline{12.011 \mathrm{~g} / \mathrm{mol}}$ | 5 | The zero is between nonzero digits - it must have been <br> measured. |
| $\underline{10200 \mathrm{~mL}}$ | 3 | The first zero is between two significant figures (so it was <br> measured), while two end zeros are placeholders. |

When doing math with measurements, you can follow two simple rules (derived from statistics)
for math operations:

1. When you multiply and divide measurements, your answer has the same number of significant figures as the measurement with the least number of significant figures. In other words, your answer is only as good as your least precise measurement!
2. When you add and subtract measurements, your answer has the same number of decimal places as the measurement with the least number of decimal places.

Let's look at a few examples:
What is the area of a rectangle with a length of 54.36 m and a width of 3.57 m ?
The area of a rectangle is equal to length times width, so

$$
54.36 \mathrm{~m} \times 3.57 \mathrm{~m}=194.0652 \mathrm{~m}^{2}
$$

The answer above was found by simply punching the numbers into a calculator. Calculators, though, don't understand the concept of significant figures and don't know that the numbers you're punching in come from measurement. We know, though, that we really don't know the area as precisely as $194.0652 \mathrm{~m}^{2}$, since we got the area by multiplying two length measurements that contained some degree of uncertainty. The proper way to report the answer is to use the multiplication / division rule. Note that 54.26 m has four significant figures while 3.57 m has three, so the final answer should have three significant figures.

$$
54.36 \mathrm{~m} \times 3.57 \mathrm{~m}=194 \mathrm{~m}^{2}
$$

The area, rounded to the correct number of significant figures, is $194 \mathrm{~m}^{2}$.
Let's try another example.
A quantity of sodium chloride is weighed in a beaker. The total weight of sodium chloride and beaker is 164.037 g , while the beaker alone weighs 15.6423 g . What is the weight of the sodium chloride?

This is a simple subtraction. Subtract the weight of the beaker from the total weight:

$$
164.037 \mathrm{~g}-15.6423 \mathrm{~g}=148.3947 \mathrm{~g} \text { sodium chloride }
$$

Again, the answer above was read straight off a calculator display. We know, though, that we don't know the weight of the sodium chloride to the precision indicated above because our total mass measurement's uncertain digit was in the thousandths place. If that digit is uncertain, we surely can't say what the digit after it is! So we use the addition / subtraction rule and round the answer to three decimal places.

$$
164.037 \mathrm{~g}-15.6423 \mathrm{~g}=\mathbf{1 4 8 . 3 9 5} \mathrm{g} \text { sodium chloride }
$$

## Rounding

In the two examples above, we rounded answers to the correct number of significant digits. The rounding rules we'll use in this class are the same as the ones you were likely taught in grade school:

1. Don't round any numbers except the result of your final calculation. This minimizes so-called "roundoff error". In short, don't round until the end.
2. If the first insignificant digit to be dropped is less than five, just drop the insignificant digits - round down.
3. If the first insignificant digit is five or greater, add 1 to the last significant digit and then drop all the insignificant digits - round up.

Let's round $\mathbf{p i}$ off to different numbers of significant figures. Pi has a value of $\mathbf{3 . 1 4 1 5 9 2 6 5}$... (it goes on, and on, and on ...)

| Significant figures | Rounded number |
| :---: | :---: |
| 3 | 3.14 |
| 4 | 3.142 |
| 5 | 3.1416 |
| 6 | 3.14159 |

For 3 and 6 significant figures, we had to round down. For 4 and 5 significant figures, we had to round up.

## Exact numbers

Some numbers are exact, which means that they contain no uncertainty. For example, a normal human has five fingers on each hand. If you pull three paper clips out of a box, you're certain that you got three - not four or two. Numbers found by counting objects are exact, and we say that they have an infinite number of significant digits.

Some other numbers are exact - like many conversion factors for units. There are exactly 100 centimeters in a meter, and exactly 12 inches in a foot. In a calculation, you'd treat these conversion factors as if they have an infinite number of significant figures.

## Summary

In this note pack, we've discussed accuracy and precision - their definitions and how they relate to experiments performed in chemical laboratories. You should know that you can get a feel for the accuracy and precision of your measurements by comparing them with known standards and by performing repeated measurements. We discussed that significant figures are a way to indicate the precision of measurements and how to report the correct number of significant figures for measurements you make in the lab. You should also know how to determine by simply looking at a number how many significant figures (and thus how precise) the number is. You should also have learned the rules for math with significant figures, rounding, and how to deal with exact numbers. These concepts are important in chemistry because chemistry depends on quantitative, reproducible measurements of chemical reactions. You'll be required to keep track of your precision in most of the labs we do in the course.

